REAL ESTATE THEORY AND MODELLING

INSTITUT DE L'ÉPARGNE IMMOBILIÈRE & FONCIÈRE

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CHRONICLE N°8

The Gordon-Shapiro model for real estate (3/4)

We showed in Chronicle 6 that Gordon-Shapiro's formula allows us to write the total return (tr) as the sum of the initial net income return (ir_1) plus the growth rate of net operating income (g) under the assumptions that the growth rate of net operating income is strictly constant over time and that we have an infinite investment horizon.

$$ir_1 = tr - g \Leftrightarrow tr = ir_1 + g$$

The infinite investment horizon hypothesis

Even if it is obvious that no one invests for an infinite period, this assumption may still be neutral with regard to the investment choices we can make? In fact, we are going to show that this hypothesis implies a stability of the total return expectation that is not verified in reality.

We will therefore begin by testing this assumption of neutrality.

To do this, we will check that neutrality is effective, i.e. that it produces the same result whatever the investment period, within the framework of the model's assumptions.

Let's imagine a 6-year investment with the following assumptions: ir_1=10, g =3% and r=5%.

Since we are looking at a 6-year investment, the general formula for today's price is as follows (see Chronicle No. 6)

$$(1)\ P_0 = \sum_{t=1}^{6} \frac{noi_t}{(1+r)^t} + \frac{P_6}{(1+r)^6}$$

with:

P_0 : the price today
P_6 : the price at the end of period 6
noi_t : the net operating income for period t
r : the discount rate

The only unknown we have is P_6.

Since we are working on the assumption that g and r are known and constant, we know P_6 (see Chronicle No. 6):

$$(2)\ P_6 = \sum_{t=1}^{\infty} \frac{noi_{t+6}}{(1+r)^t}$$

What can still be written (see Chronicle No. 6):

$$(3)\ P_0 = \frac{noi_7}{r-g}$$

Let's take a look at the results using a small model created in Excel.

Table 1

t	noi	g	r	$(1+r)^t$	$noi_t/(1+r)^t$	P_0 (g constant)
1	10.0		5%	1.05	9.52	9.52
2	10.3	3%	5%	1.10	9.34	18.87
3	10.6	3%	5%	1.16	9.16	28.03
4	10.9	3%	5%	1.22	8.99	37.02
5	11.3	3%	5%	1.28	8.82	45.84
6	11.6	3%	5%	1.34	8.65	54.49
7	11.9	3%	5%	1.41	8.49	62.98
...	...	...	...	...	...	...

So we find that noi_7=11.9... and that $\sum_{t=1}^{6} \frac{noi_t}{(1+r)^t} = 54.49\ ...$

If I replace the variables in equation (3) with the values calculated in my example, I find:

$$P_6 = \frac{11.9\ ...}{5\% + 3\%} = 597.0\ ...$$

If I replace the variables in equation (1) with these same values, I obviously find:

$$P_0 = 54.49\ ... + \frac{597.0\ ...}{(1+5\%)^6} = 500$$

In other words, the same value of P_0 as if I were investing for an infinite period from period 0, the result of which we have already calculated in Chronicle 7:

$$P_0 = \frac{noi_1}{r+g} = \frac{10}{5\% - 3\%} = 500$$

And so, under the assumptions of the model, with *r* and *g* constant, and because it is an equilibrium model, the price today of any asset is totally indifferent to the length of time it is held. I can hold it for 3, 6, 9, 12 years or for life, and the price is the same. The assumption of an infinite investment period is neutral.

Now let's think a little about what this infinite horizon assumption actually implies.

In addition to the assumption that the rate of growth of rental income will be constant over time, already discussed in Chronicle No. 7, this assumption also implies that the discount rate I am inclined to use in period 0, at the time of purchase, must be the same as the one I will wish to use in period 6 at the time of resale.

In other words, at the time of purchase I make a calculation taking into account the equilibrium discount rate, which is 'normal' in the context of the period of purchase and, for the equilibrium model to work, it is also necessary, 6 years, 9 years or even many more years later, for the discount rate used at the time of resale to be the same as that used at the time of purchase. This is a classic assumption in equilibrium models, but is it realistic?

Because if it isn't then, in our example, P_6 is no longer equal to 597.0... and so there is no longer any equivalence between the assumption of an infinite horizon and the reality of an investment with a finite duration (in our example 6 years).

Let's imagine, for example, that in six years the discount rate has not remained stable but has fallen slightly from 5% to 4.8%. (5% for the first 6 years, then 4.8% thereafter).

In this case, we always find that noi_7=11.9... and that $\sum_{t=1}^{6} \frac{noi_t}{(1+r)^t} = 54.49...$

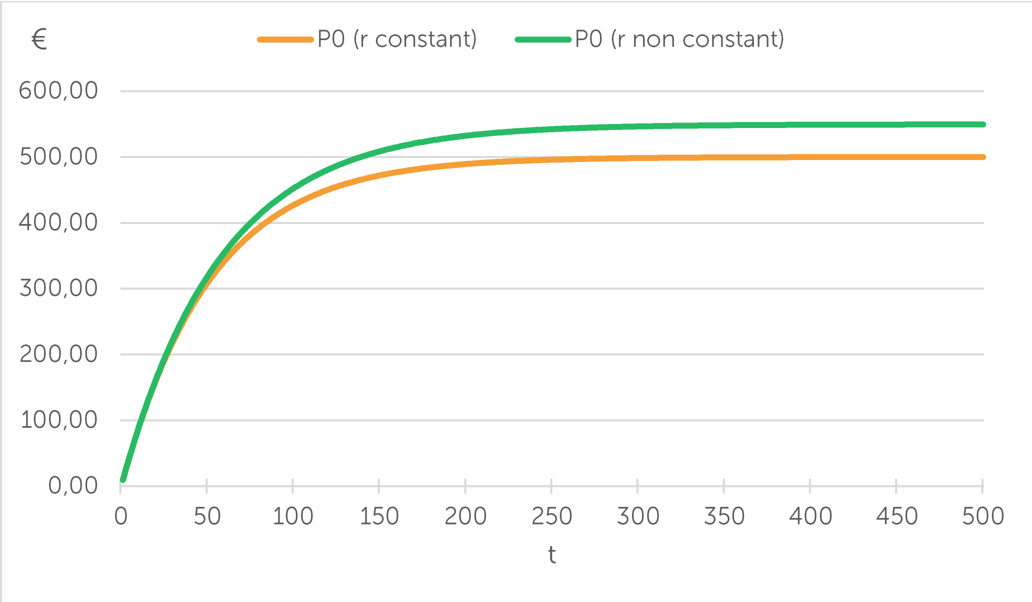
If I replace the variables in equation (3) with the values calculated in my new example, I find:

$$P_6 = \frac{11.9\ ...}{4.8\% + 3\%} = 663.3\ ...$$

If I replace the variables in equation (1) with these new values, as before, this time I find:

$$P_0 = \ 54.49\ ... + \frac{663.3\ ...}{(1 + 5\%)^6} = 549.5$$

In other words, the value of P_0 differs by around 10% compared with the assumption that the discount rate remains constant.



Let's now look at what the constant discount rate assumption means.

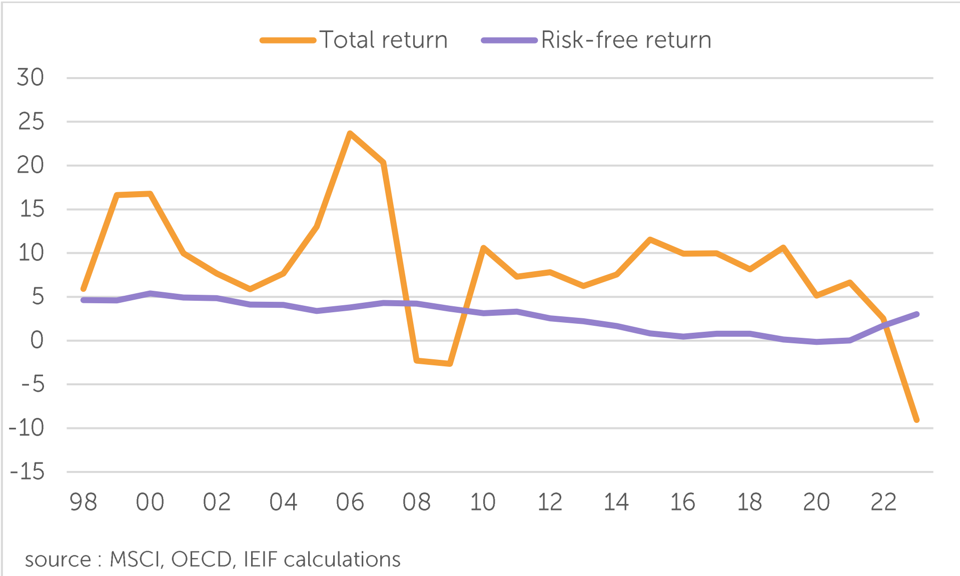
We know that this discount rate *r* must be equal to the desired/required total return for the asset analysed, i.e. it must be equal to the sum of the expected risk-free rate plus the expected desired/required risk premium.

$$r = trex = E(rfr) + E(\pi)$$

with:
- $E(\ldots)$: expectation (the average)
- *trex* : expected total return
- *rfr* : risk-free return
- π : risk premium

The constancy of *r* therefore implies that either the risk-free return and the property risk premium remain constant in expectation, or that the variation in one is offset by the variation in the other.

Let's look at the historical trends in these data for the periods from 1998 to 2023 for the Paris office market.



Let's just carry out a very simple exercise, arbitrarily dividing the period into two 13-year periods: 1998-2010 and 2011-2023.

Table 2: Geometric mean over 13 years

	1998-2010	2011-2023
Total return	10.0%	6.4%
Risk-free return	4.3%	1.4%
Risk premium	5.7%	5.0%

source: MSCI, OECD, IEIF calculations

As can be seen from this example, *r* is clearly not constant: the average total return observed fell over the period studied, from 10% at the start of the period (98-10) to 6.4% at the end (11-23). The overall return follows the downward trend in the risk-free return, while the property risk premium shows little decline.

The most important lesson is that **the discount rate may not be stable over time, making the simplifying assumption of an infinite time horizon irrelevant.**

In the next Chronicle, we will bring the simplified Gordon-Shapiro model into line with the general definition studied in the first five Chronicles.

These chronicles are linked to my activity at the IEIF. a Paris based think tank on real estate where I conduct research into the modelling of major property variables.
For those less familiar with property analysis. these chronicles can be a source of information and a knowledge base. For experts in the field. their purpose is to launch discussions and exchanges on the various subjects I cover.
Some of the chronicles will be based on known and familiar elements. while others will deal with research elements and present some of the results of my work.